Int. J. Solids Structures Vol. 24, No. 10, pp. 1045–1057, 1988
Printed in Great Britain.
0020–7683/88 $3.00+ .00
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A DAMAGE MECHANICS MODEL FOR ANISOTROPIC MATERIAL AND ITS APPLICATION TO SHEET METAL FORMING

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(*Received* 6 *May* 1987)

Abstract—An isotropic plastic damage model for orthotropic materials is proposed from continuum damage theory, and is simplified for a transversely isotropic material in the plane stress state. The simplified damage model is compared with the relative experiments and is proved to be applicable to sheet metal forming. By using the present damage equations combined with the M–K model in this paper, the effects of internal damage, anisotropy of material and the initial inhomogeneities of material properties on the limit strains in a biaxially stretching sheet are investigated.

1. INTRODUCTION

Many investigations have shown that ductile fracture in metals involves considerable plastic damage via the nucleation, growth and coalescence of microvoids. These voids nucleate mainly at second phase particles by particle fracture or by interfacial decohesion, and subsequently they grow further with an increase of plastic deformation. When the ligaments between neighbouring voids become thinner and thinner, the true stress in the ligaments becomes greater and greater. Then the internal necking between neighbouring voids takes place and ductile fracture occurs.

In the case of sheet materials, many theoretical and experimental studies have also shown that the internal damage plays a significant role in triggering of localized necking in biaxially stretching sheets. A lot of effort has gone in to correlating the limit strains of a sheet with microstructural weaknesses such as voids and inclusions. Needleman and Triantafyllidis (1978) and Chu and Needleman (1982) used Gurson's constitutive model of a porous plastic solid (Gurson, 1977) to characterize the sheet material. Parmar and Mellor (1980), Thomson and Nayak (1980) and Jalinier and Schmitt (1982) experimentally studied the void growth in sheet metal during deformation. Kim and Kim (1983) investigated the effect of void growth on the limit strains in sheet metal stretching based on Parmar and Mellor's experimental results on void growth, which show that the growth of internal voids has a great effect on the limit strains of sheets subject to biaxial tension.

In 1983, Lemaitre, first applied the continuum damage mechanics concept to sheet metal forming and obtained the fracture limit curves of a sheet. After that Lee *et al.* (1983, 1985) proposed a plastic damage model and a plastic damage instability criterion to predict the limit strains in sheet metal. But the results showed a deviation from the experimental data in the biaxial state.

In this paper, an isotropic ductile plastic damage model for an orthotropic material was proposed based on the effective stress concept, on the hypothesis of strain equivalence and on thermodynamics. For a transversely isotropic material in the plane stress state, a simplified model can be derived and can be used for sheet metal forming. A comparison of the theoretical model and experimental results given by Parmar and Mellor (1980) and Jalinier and Schmitt (1982) was made and a qualitative agreement was found. Also, the application of the plastic damage equation combined with the M–K model to a sheet in biaxially stretching shows that a conventional over-estimation of limit strains can be significantly reduced by taking the internal damage into consideration.

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2. DAMAGE MODEL FOR ORTHOTROPIC MATERIALS

2.1. *Damage variable D*

In general, the damage process is regarded as the generation and growth of microdefects (microvoids and microcracks) within an initially perfect material. The material remains the same but its macroscopic properties change with its microscopic geometry. The damage variable D is a macroscopic measure of the microscopic geometrical deterioration of the material.

Consider a volume element at macroscale in a damaged body, that is of a size large enough to contain many defects, and small enough to be considered as a material point of the mechanics of continua. Let A_0 be the overall section area of this element, defined by the normal **n**, and A_{eff}, the effective resisting area (i.e. A_0 diminished from the surface intersections of the microvoids). By definition, the damage variable D associated to the direction of the normal **n** is (Lemaitre, 1984)

$$D_n = \frac{A_0 - A_{eff}}{A_0} \tag{1}$$

which is identical with the void area fraction in the section defined by the normal **n**.

In the case of ductile plastic damage of metals, the internal damage is usually considered to be caused mainly by the nucleation, growth and coalescence of microvoids and may be assumed approximately to be isotropic up to the critical moment of plastic instability even the material properties are anisotropic. Thus we restrict ourselves to isotropic damage and characterize the damage of orthotropic materials with a scalar D.

Furthermore, the damage variable may be expressed in terms of the variation of the elasticity modulus as follows (Lemaitre, 1984):

$$D = 1 - \bar{E}/E \tag{2}$$

where E is Young's modulus of undamaged material and $\bar{E}$ can be considered as the elasticity modulus of the damaged material.

The effective stress (or net stress) is

$$\sigma^*_{ij} = \sigma_{ij}/(1-D). \tag{3}$$

2.2. *Damage constitutive equation*

Taking the free-energy ψ as the thermodynamic potential, it is assumed that it is a convex function of all observable and internal variables. Let E be the fourth-order tensor of elasticity and ρ the density; then

$$\psi_e = \tfrac{1}{2}E_{ijkl}\varepsilon^e_{ij}\varepsilon^e_{kl}(1-D). \tag{4}$$

The damaged elasticity law is

$$\sigma_{ij} = \rho\frac{\partial\psi_e}{\partial\varepsilon^e_{ij}} = E_{ijkl}\varepsilon^e_{kl}(1-D) \tag{5}$$

and the damage strain energy release rate Y is defined by

$$Y = \rho\frac{\partial\psi_e}{\partial D} = -\tfrac{1}{2}E_{ijkl}\varepsilon^e_{ij}\varepsilon^e_{kl}. \tag{6}$$

Suppose the principal stresses act along the principal axes of orthotropy which do not change during deformation. Using the principal stresses σ_i and strains ε_i ($i = 1, 2, 3$) instead of the corresponding stress components σ_{ij} and strain components ε_{ij} ($i, j = 1, 2, 3$), we can express the damage strain release rate as follows:

$$Y = \frac{-1}{2(1-D)^2}\left[\frac{\sigma_1^2}{E_1}+\frac{\sigma_2^2}{E_2}+\frac{\sigma_3^2}{E_3}-\frac{2\nu_{12}}{E_1}\sigma_1\sigma_2-\frac{2\nu_{23}}{E_2}\sigma_2\sigma_3-\frac{2\nu_{31}}{E_3}\sigma_3\sigma_1\right] \tag{7}$$

where E_i ($i = 1, 2, 3$) is the elastic modulus in the direction of the principal axis i, and ν_{ij} is Poisson's ratio in the principal direction j when the stress acts along the principal axis i and stresses in other directions vanish; that is

$$\nu_{ij} = -\varepsilon_j/\varepsilon_i, \quad \nu_{ij}/E_i = \nu_{ji}/E_j. \tag{8}$$

A dissipation potential is chosen as follows:

$$\varphi^*(Y, p, D, T) = \tfrac{1}{2}\cdot S_0(-Y/S_0)^2 D\cdot \dot{p}. \tag{9}$$

The complementary law of evolution of damage derived from φ^* by

$$\dot{D} = -\partial\varphi^*/\partial Y = \frac{Y}{S_0}D\dot{p} \tag{10}$$

where S_0 is a material constant and p the accumulated plastic strain defined by

$$\dot{p} = \frac{1}{GH+HF+FG}\{\tfrac{2}{3}(F+G+H)[F(G\dot{\varepsilon}_2-H\dot{\varepsilon}_3)^2+G(H\dot{\varepsilon}_3-F\dot{\varepsilon}_1)^2+H(F\dot{\varepsilon}_1-G\dot{\varepsilon}_2)^2]\}^{1/2}. \tag{11}$$

According to Lemaitre's hypothesis of strain equivalence the Ramberg–Osgood hardening law coupled with damage should be written as follows:

$$K\cdot p^n = \frac{\bar{\sigma}}{1-D} \tag{12}$$

where K and n are material constants, and $\bar{\sigma}$ the effective stress

$$\bar{\sigma} = \left\{\frac{3}{2(F+H+G)}[F(\sigma_2-\sigma_3)^2+G(\sigma_3-\sigma_1)^2+H(\sigma_1-\sigma_2)^2]\right\}^{1/2}. \tag{13}$$

Using eqns (7), (10) and (12), we obtain the damage constitutive equation for orthotropic material

$$\mathrm{d}D/D = A\cdot F(\sigma_i/\bar{\sigma})\cdot p^{2n}\,\mathrm{d}p \tag{14}$$

in which A is a material constant

$$A = \frac{K^2}{2E_1S_0}$$

and

$$F(\sigma_i/\bar{\sigma}) = \left(\frac{\sigma_1}{\bar{\sigma}}\right)^2+\frac{E_1}{E_2}\left(\frac{\sigma_2}{\bar{\sigma}}\right)^2+\frac{E_1}{E_3}\left(\frac{\sigma_3}{\bar{\sigma}}\right)^2-2\nu_{12}\left(\frac{\sigma_1}{\bar{\sigma}}\right)\left(\frac{\sigma_2}{\bar{\sigma}}\right)$$
$$-2\nu_{23}\left(\frac{\sigma_2}{\bar{\sigma}}\right)\left(\frac{\sigma_3}{\bar{\sigma}}\right)\frac{E_1}{E_2}-2\nu_{31}\left(\frac{\sigma_3}{\bar{\sigma}}\right)\left(\frac{\sigma_1}{\bar{\sigma}}\right)\frac{E_1}{E_3}. \tag{15}$$

In the case of proportional loading the ratio $\sigma_i/\bar{\sigma}$ = const. and eqn (14) can be integrated under the initial condition

$$D_{p=p_0} = D_0.$$

So we have

$$\ln \frac{D}{D_0} = A \cdot F(\sigma_i/\bar{\sigma}) \left(\frac{1}{2n+1}\right)(p^m - p_0^m) \tag{16}$$

or

$$D = D_0 \exp\,[A \cdot F(\sigma_i/\bar{\sigma})(2n+1)^{-1}(p^m - p_0^m)] \tag{17}$$

where

$$m = 2n+1.$$

If the rupture strain p_c corresponds to the intrinsic value of damage at failure D_c, which is assumed to be a material property, i.e. $D = D_c$ as $p = p_c$, then

$$D = D_0 \cdot \exp\left[\ln \frac{D_c}{D_0}\left(\frac{p^m - p_0^m}{p_c^m - p_0^m}\right)\right]. \tag{18}$$

In the particular case of one-dimensional $D = D_0 \leftrightarrow p = \varepsilon_0$ and $D = D_c \leftrightarrow p = \varepsilon_c$, which leads to

$$A = \left[\frac{3(G+H)}{2(F+G+H)}\right]^{1+m/2} m \frac{1}{\varepsilon_c^m - \varepsilon_0^m} \ln\left(\frac{D_c}{D_0}\right). \tag{19}$$

For a perfectly plastic material, $m = 1$, we have

$$\mathrm{d}D/D = A \cdot F(\sigma_i/\bar{\sigma})\ \mathrm{d}p. \tag{20}$$

In the case of proportional loading, the damage equation is

$$D = D_0 \cdot \exp\,[A \cdot F(\sigma_i/\bar{\sigma})(p - p_0)] \tag{21}$$

where

$$A = \left[\frac{3(G+H)}{2(F+G+H)}\right]^{3/2} \frac{1}{\varepsilon_c - \varepsilon_0} \ln\,(D_c/D_0). \tag{22}$$

2.3. *Damage criterion for ductile fracture*

We assume that ductile fracture would occur if the damage parameter $\bar{V}_D$ reaches its critical value $\bar{V}_{D_c}$ in an anisotropic material, then we obtain a damage failure criterion for orthotropic material:

$$\bar{V}_D = A^{-1} \int_{D_0}^{D_c} \mathrm{d}D/D = \bar{V}_{D_c}. \tag{23}$$

Noting that $p = \bar{\varepsilon}_p$, where $\bar{\varepsilon}_p$ is the effective plastic strain, then from eqn (20) we have

$$\bar{V}_D = \int_{\bar{\varepsilon}_0}^{\bar{\varepsilon}_c} F(\sigma_i/\bar{\sigma})\ \mathrm{d}\bar{\varepsilon}_p = \bar{V}_{D_c} \tag{24}$$

or

$$\bar{V}_D = F(\sigma_i/\bar{\sigma})(\bar{\varepsilon}_p - \bar{\varepsilon}_0) = \bar{V}_{D_c} \tag{25}$$

for proportional loading, where $\bar{V}_{D_c} = A^{-1} \ln (D_c/D_0)$, is a critical damage parameter, which can be reasonably regarded as a material property.

It is easy to see that the present criterion (24) is equivalent to the plastic strain rupture criterion under radial loading

$$\bar{\varepsilon}_p = \bar{\varepsilon}_c \tag{26}$$

and to Lemaitre's continuum damage mechanics criterion

$$D = D_c. \tag{27}$$

For an isotropic material, damage criterion (24) is simplified as follows:

$$V_D = \int_{\bar{\varepsilon}_0}^{\bar{\varepsilon}_c} f(\sigma_m/\bar{\sigma})\, \mathrm{d}\bar{\varepsilon}_p = V_{D_c} \tag{28}$$

where

$$f(\sigma_m/\bar{\sigma}) = \tfrac{2}{3}\cdot(1+\nu) + 3(\sigma_m/\bar{\sigma})^2(1-2\nu) \tag{29}$$

which is the same as that given previously (Tai and Yang, 1987).

3. PLASTIC DAMAGE EQUATION APPLICABLE TO SHEET METAL FORMING

In general, normal anisotropy and planar isotropy is assumed in the analysis of sheet metal forming, so that $E_1 = E_2 = E$, and $\nu_{12} = \nu$. Also, the loading condition is reasonably thought of as the plane stress state during deformation of the sheet; $\sigma_3 = 0$. Thus in eqn (14)

$$F(\sigma_i/\bar{\sigma}) = \left(\frac{\sigma_1}{\bar{\sigma}}\right)^2 + \left(\frac{\sigma_2}{\bar{\sigma}}\right)^2 - 2\nu\left(\frac{\sigma_1}{\bar{\sigma}}\right)\left(\frac{\sigma_2}{\bar{\sigma}}\right). \tag{30}$$

From Hill's theory of anisotropic plasticity (Hill, 1950) we can express the damage equation for transversely isotropic material as follows:

$$D = D_0 \exp\,[B\cdot F_1(\alpha)(p^m - p_0^m)] \tag{31}$$

where

$$B = \frac{1}{\varepsilon_c^m - \varepsilon_0^m} \ln \frac{D_c}{D_0} \tag{32}$$

$$F_1(\alpha) = \frac{1+\alpha^2-2\nu\alpha}{1+\alpha^2-\beta\alpha}\left[\frac{3(1+r)}{2(2+r)}\right]^{m/2} \tag{33}$$

$$\bar{\sigma} = \sqrt{\left(\frac{3}{2}\right)}\left\{\frac{1}{2+r}[\sigma_1^2+\sigma_2^2+r(\sigma_1-\sigma_2)^2]\right\}^{1/2} \tag{34}$$

$$\dot{p} = \sqrt{\left(\frac{3}{2}\right)}\frac{\sqrt{(2+r)}}{1+2r}[(\dot{\varepsilon}_2 - r\dot{\varepsilon}_3)^2 + (\dot{\varepsilon}_1 - r\dot{\varepsilon}_3)^2 + r(\dot{\varepsilon}_1-\dot{\varepsilon}_2)^2]^{1/2} \tag{35}$$

$$\beta = \frac{2r}{1+r}$$

$$\alpha = \sigma_2/\sigma_1. \tag{36}$$

Noting $p = \bar{\varepsilon}_p$, where $\bar{\varepsilon}_p$ is the effective plastic strain, the damage equation, eqn (31), takes the following form:

$$D = D_0 \exp\,[B \cdot F_1(\alpha)(\bar{\varepsilon}_p^m - \bar{\varepsilon}_0^m)]. \tag{37}$$

In the case of perfect plasticity, we have

$$D = D_0 \exp\,[C \cdot F_1(\alpha)(\varepsilon_p - \varepsilon_0)] \tag{38}$$

where

$$C = \frac{1}{\varepsilon_c - \varepsilon_0} \ln\,(D_c/D_0). \tag{39}$$

For simplification, we restrict ourselves to the perfectly plastic material and discuss eqn (38) only. A similar analysis can be used to damage equation (37).

From eqn (38) we can see that the internal damage of material depends not only on the material properties but also on the stress state and plastic strain. Also the initial damage and initial strain have great effects on the development of damage.

Using the associated flow rule, we have

$$\alpha = \frac{(1+r)\rho + r}{1+r+r\rho}. \tag{40}$$

Substituting eqn (40) into eqn (38), we obtain

$$D = D_0 \exp\,[\bar{C} f(\rho)(\bar{\varepsilon}_p - \bar{\varepsilon}_0)] \tag{41}$$

where

$$f(\rho) = \frac{1+\rho^2+R\rho}{1+\rho^2+\beta\rho}$$

$$R = \frac{4r(1+r) - 2\nu[(1+r)^2+r^2]}{(1+r)^2+r^2-2\nu r(1+r)} \tag{42}$$

and $\bar{C}$ is a material-dependent constant

$$\bar{C} = \frac{(1+r)^2+r^2-2\nu r(1+r)}{1+2r}\left[\frac{3(1+r)}{2(2+r)}\right]^{1/2} C. \tag{43}$$

Function $f(\rho)$ is related to Poisson's ratio ν, the coefficient of normal anisotropy r and the ratio of the principal strains, ρ, and changes greatly with an increase of the strain ratio under uniaxial loading. The greater the r-value and Poisson's ratio ν are, the greater the

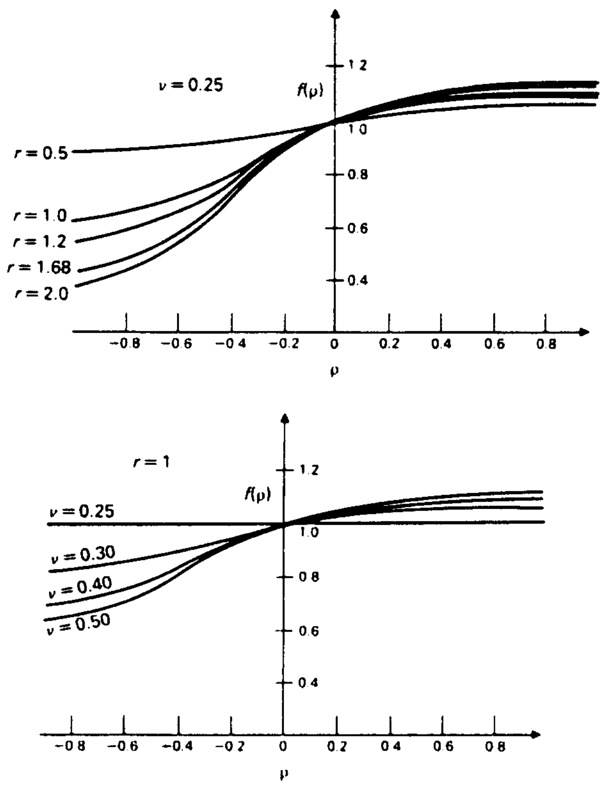


Fig. 1. The function $f(\rho)$ for various $r(a)$ and $\nu(b)$.

change of $f(\rho)$ is. But under biaxial loading, function $f(\rho)$ is weakly dependent upon the strain ratio ρ, and may be considered to be independent of the strain path (Fig. 1); that is

$$D = D_0 \exp\left[K_0 \cdot (\bar{\varepsilon}_p - \bar{\varepsilon}_0)\right] \tag{44}$$

which is in agreement with Parmar and Mellor's experimental results (Parmar and Mellor, 1980).

Using eqn (35), we can also reduce the damage equation, eqn (41), into the following form:

$$D = D_0 \exp\left[H \cdot g(\rho)(\varepsilon_1 - \varepsilon_0)\right] \tag{45}$$

where

$$g(\rho) = \frac{1+\rho^2+R\rho}{(1+\rho^2+\beta\rho)^{1/2}} \tag{46}$$

$$H = \left[\frac{2(1+r)(2+r)}{3(1+2r)}\right]^{1/2} \bar{C}. \tag{47}$$

The values of $g(\rho)$ are plotted against strain ratios ρ for different Poisson's ratios ν and r-values in Figs 2(a) and (b), respectively, from which it is easy to see that $g(\rho)$ is approximately linear for the range $-0.4 \leqslant \rho \leqslant 1$. But for a large r-value and small ν-value, $g(\rho)$ slightly deviates from linearity; which agrees with Jalinier and Schmitt's analysis (Jalinier and Schmitt, 1982).

In fact, an analytical linearization of function $g(\rho)$ should give

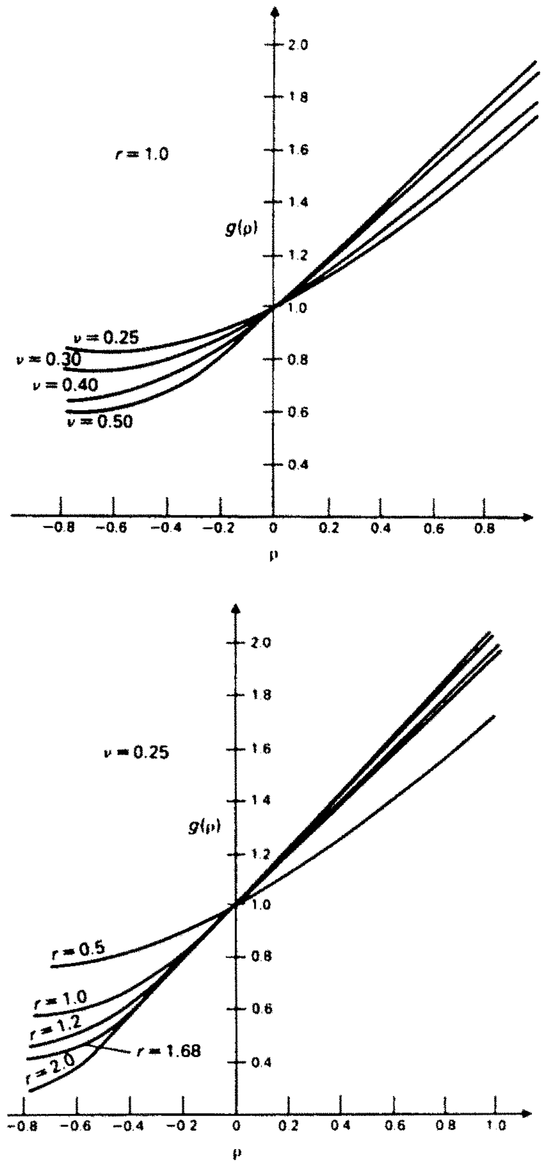


Fig. 2. The function $g(\rho)$ for various $\nu(a)$ and $r(b)$.

$$g(\rho) = 1 + b\rho \tag{48}$$

where

$$b = \frac{3r(1+2r)+2r^2-2\nu(1+r)^3}{(1+r)^3+r^2(1+r)-2\nu r(1+r)^2}.$$

Its deviation from eqn (46) is less than 13.2% for the range of $-0.4 \leqslant \rho \leqslant 1$ and $0 \leqslant \nu \leqslant 0.4$.

For an isotropic material

$$b = \frac{11-16\nu}{10-8\nu}.$$

The deviation due to linearization is less than 10% for $0 \leqslant \nu \leqslant 0.4$.

In eqn (48), if we approximately let $b \approx 1$, then we can obtain an equation similar to that of Jalinier and Schmitt (1982)

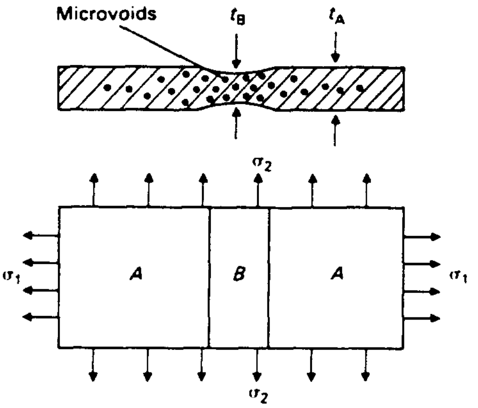


Fig. 3. The modified M–K model.

$$D = D_0 \exp\left[H(1+\rho)(\varepsilon_1-\varepsilon_0)\right] \tag{49}$$

which is also in coincidence with Parmar and Mellor's experimental observation.

4. APPLICATIONS

In this section, the above plastic damage equations combined with a modified M–K model will be used to analyze the limit strains in a biaxially stretching sheet.

As in the original analysis of Marciniak and Kuczynski (1967), an inhomogeneous band is considered, region B in Fig. 3, in an otherwise homogeneous sheet. It is assumed that outside the band, region A in Fig. 3, homogeneous and proportional straining is maintained. Plane stress conditions are assumed to prevail both inside and outside the band. Also the inhomogeneity is assumed to exist which develops into a neck after the effective plastic strain $\bar{\varepsilon}_p$ reaches the critical threshold $\bar{\varepsilon}_0$, and this inhomogeneity is both a geometrical inhomogeneity arising from the unequal thickness of the sheet metal or from the roughness of its surface and a physical inhomogeneity caused by an inhomogeneous distribution of impurities and voids. As necking develops, the total inhomogeneity rapidly increases and the equilibrium condition between regions A and B may be expressed as follows:

$$\sigma_{1A} t_A (1-D_A) = \sigma_{1B} t_B (1-D_B) \tag{50}$$

where t is the thickness of a sheet and Marciniak and Kuczynski's inhomogeneity indicating the variation of the sheet thickness is defined by

$$f = t_B/t_A. \tag{51}$$

Here, and subsequently, subscripts $(\ )_A$ and $(\ )_B$ refer to quantities outside and inside the neck, respectively.

By using the damage equation, eqn (38), and eqn (50), an analysis similar to that given by Marciniak and Kuczynski (1967) will lead to the following equation for $x(=\bar{\varepsilon}_B-\bar{\varepsilon}_0)$ and $y(=\bar{\varepsilon}_A-\bar{\varepsilon}_0)$:

$$\frac{\sqrt{(1-B^2)}}{\sqrt{(A^2-B^2y'^2)}} = f_0 \frac{K_B}{K_A}\frac{(x+\bar{\varepsilon}_0)^{n_B}}{(y+\bar{\varepsilon}_0)^{n_A}}\frac{1-D_{B0}\exp\left[\bar{C}_B f_B(\rho)\cdot x\right]}{1-D_{A0}\exp\left[\bar{C}_A f_A(\rho)\cdot y\right]} \times \exp\left[Py - Q\int_0^x \sqrt{(A^2-B^2y'^2)}\,\mathrm{d}x\right] \tag{52}$$

where

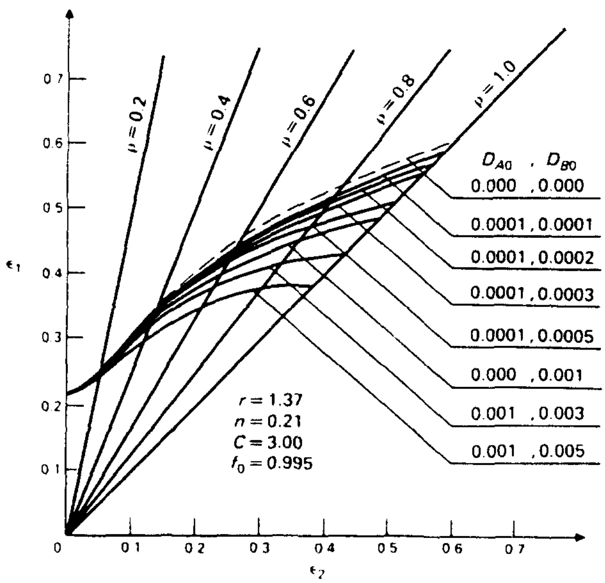


Fig. 4. Effects of initial damage on the limit strains.

$$A = \frac{2}{3}\frac{2+r}{1+r} \tag{53}$$

$$B = \frac{(1+2r)^{1/2}}{1+r}\frac{\rho}{(1+\rho^2+\beta\rho)^{1/2}} \tag{54}$$

$$P = \sqrt{\left(\frac{3}{2}\frac{1+2r}{(1+r)(2+r)}\right)}\frac{1+1/2\beta\rho}{(1+\rho^2+\beta\rho)^{1/2}} \tag{55}$$

$$Q = \sqrt{\left(\frac{3}{2}\frac{1+2r}{(1+r)(2+r)}\right)}. \tag{56}$$

For a set of values of n_A, n_B, K_A, K_B, C_A, C_B, D_{A0}, D_{B0}, f_0, $\bar{\varepsilon}_0$ and r, eqn (52) can be solved numerically by the Runge–Kutta method under the following initial conditions:

$$x = 0,\quad y = 0,\quad y' = B^{-1}\sqrt{(A^2-(1-B^2)/G^2)} \tag{57}$$

where

$$G = f_0\frac{K_B}{K_A}\bar{\varepsilon}_0^{n_A-n_B}\frac{1-D_{B0}}{1-D_{A0}}.$$

It is easy to see that the value of y' decreases with an increase of x and y, and local necking takes place when y' approaches zero. In calculations we choose 1.5×10^{-3} as the convergence limit and take the strain values at this state as the limit strains.

Here, we mainly consider the effect of inhomogeneity of internal damage. At the same time, we also consider the effects of inhomogeneities in the form of differences of material properties. For this reason, we choose

$$r = 1.37,\quad \nu = 0.3,\quad n_A = 0.21,\quad K_A = 1,\quad C_A = 3,\quad f_A(\rho) \approx f_B(\rho)$$

and

$$\bar{\varepsilon}_0 = \bar{\varepsilon}_i = \sqrt{\left(\frac{2(2+r)}{3(1+r)}\right)}\frac{4n_A(1-\beta\alpha+\alpha^2)^{3/2}}{(2-\alpha\beta)^2+\alpha(2\alpha-\beta)^2} \tag{46}$$

in which $\bar{\varepsilon}_i$ is the strain at the Swift instability (Swift, 1952).

Figure 4 displays the forming limit curves for the case where $f_0 = 0.995$, the initial damage takes various combinations of D_{A0} and D_{B0} and all other quantities are identical

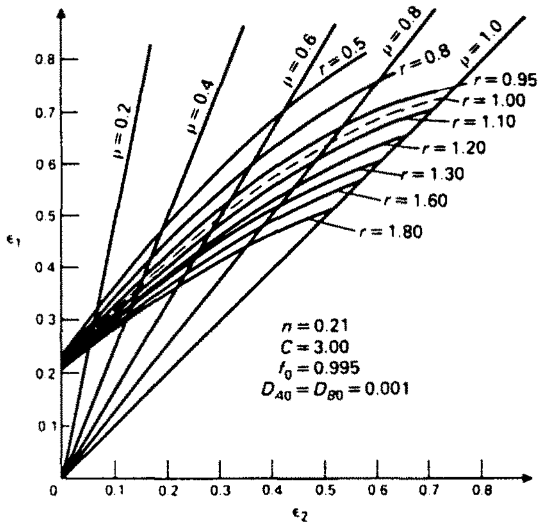


Fig. 5. Effects of anisotropy of material on the limit strains.

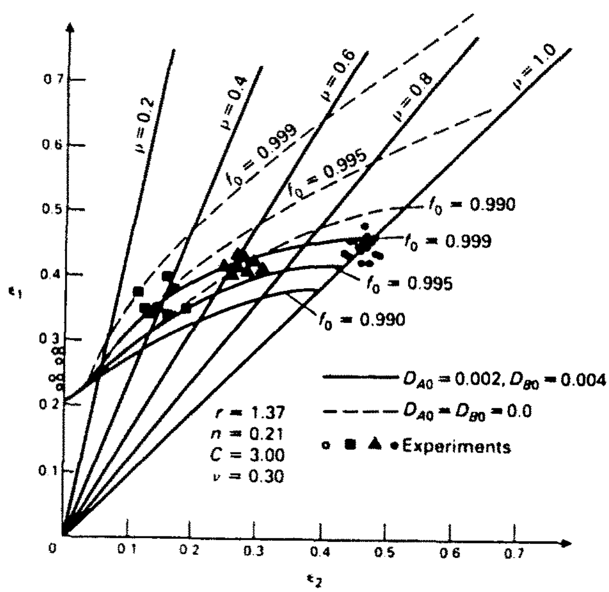


Fig. 6. Forming limit curves for rimming steel.

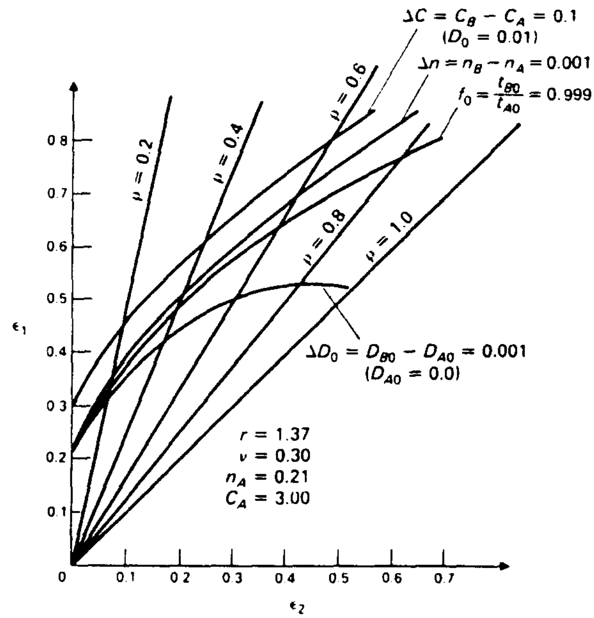
in regions A and B. It is clear that the internal damage has a significant influence on the limit strains, and the inhomogeneous distribution of the internal damage plays a greater role than the damage itself if the other factors remain constant.

Figure 5 shows the effect of anisotropy of materials on the limit strains of the sheet, where $f_0 = 0.995$ and $D_{A0} = D_{B0} = 0.001$, from which we may see that the limit strains decrease with an increase of the coefficient of normal anisotropy r and the effect of the anisotropy of material is also significant. Thus in the analysis of sheet metal forming it is important to consider the effect of anisotropy.

Figure 6 shows the results of calculations compared with the experimental results of Tadros and Mellor (1978) for rimming steel, in which $D_{A0} = 0.002$, $D_{B0} = 0.004$ and $f_0 = 0.999$, 0.995, 0.990. It is evident that a better correlation can be found between the calculation and experiment when the value of f_0 lies between 0.995 and 0.999. This range of values for f_0 seems reasonable and the combination of D_{A0} and D_{B0} is also acceptable.

Figures 7(a) and (b) show the forming limit curves resulting from various types of inhomogeneities. The curves marked $\Delta n = n_B - n_A$, correspond to a difference of the strain-hardening exponent between regions B and A, with all other initial values being identical

(a)



(b)

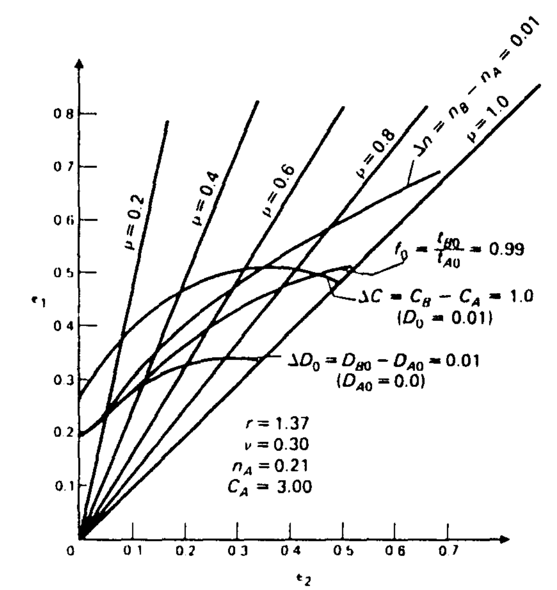


Fig. 7(a, b). The calculated forming limit curves resulting from various types of initial inhomogeneities.

in regions A and B. Similarly, the curves marked $\Delta C = C_B - C_A$, correspond to a difference of the material constant C (C is given by eqn (39)) between regions B and A, with $D_{A0} = D_{B0} = 0.01$ and all other initial values being identical in regions A and B and so on. It is seen that the inhomogeneities in the form of material properties (n, K, C and D) give qualitatively the same behaviour as the initial thickness inhomogeneity f_0.

5. CONCLUSIONS

This paper presents an isotropic plastic damage mode for an orthotropic material and a simplified damage model applicable to the analysis of sheet metal forming limits. From the model we know that the internal damage of a sheet subject to tension depends not only on material properties such as the coefficient of normal anisotropy r, Poisson's ratio ν, etc. but also on the effective plastic strain and stress state. A close examination of damage equations shows that in the case of biaxial tension the internal damage may be approximately thought to be independent of stress state and dependent only on material properties and plastic strain, which is in accordance with the experimental observations. Application of the present damage equation combined with the modified M–K model shows that the internal damage has a significant influence on the limit strains and the inhomogeneities of material properties such as n, K, C, D, etc. have a similar effect as the initial thickness inhomogeneity in a biaxially stretching sheet.

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